Dimensional analysis of natural debris flows

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Abstract: Debris flow is generally composed of a wide range of solid particles and viscous pore fluid. It flows at a high velocity down a slope channel. Interactions between solid and fluid phases affected by multiple parameters govern the rheological properties of debris flows. A dimensional analysis for a systematic study of the governing parameters is presented. Multiple dimensionless numbers with clear physical meanings are critically reviewed. The applications of field data to the studying of natural debris flows are demonstrated. Specific values of dimensionless numbers for classifying flow regimes of large-scale natural debris flows are obtained. Compared with previous small-scale physical model tests, this study shows that the contact friction between particles dominates in natural debris flows. In addition, the solid inertial stress due to particle collisions and the pore-fluid viscous shearing stress play key roles in governing the dynamic properties of debris flows. The channel width as a confinement to the flows can affect the solids discharge per unit width significantly. A dimensionless number related to pore-fluid pressure dissipation is also found that can satisfactorily distinguish between surge flows and continuous flows in the field.

Key words: debris flow, dimensional analysis, dimensionless number, flow regime, channel confinement, classification.

Résumé : Un écoulement de débris est généralement composé de particules solides et d'un fluide interstitiel visqueux, et se déplace à grande vitesse en suivant des canaux en pente. Les propriétés rhéologiques des écoulements de débris sont gouvernées par les interactions entre les phases fluides et solides, qui elles sont affectées par de multiples paramètres. L'article présente une étude systématique des paramètres par analyse dimensionnelle. Plusieurs nombres sans dimension ayant des correspondances physiques bien établies sont révisés. L'application de données de terrain lors de l'étude d'écoulements de débris naturels est démontrée. Des valeurs spécifiques ont été obtenues pour les nombres sans dimension utilisés pour classifier les régimes d'écoulement des écoulements de débris naturels à grande échelle. Comparativement aux modèles physiques précédents sur une petite échelle, cette étude montre que la friction de contact entre les particules domine dans le cas d'écoulements de débris naturels. De plus, la contrainte d'inertie du solide due aux collisions entre les particules, et la contrainte de cisaillement visqueuse du fluide interstitiel, jouent des rôles importants dans l'établissement des propriétés dynamiques des écoulements de débris. La largeur du canal qui confine l'écoulement peut affecter de façon significative la décharge de solides par unité de largeur. Un nombre sans dimension relié à la dissipation des pressions des fluides interstitiels a aussi été déterminé afin de distinguer correctement sur le terrain les écoulements en surpression des écoulements continus.

Mots-clés : écoulement de débris, analyse dimensionnelle, nombre sans dimension, régime d'écoulement, confinement par un canal, classification.

[Traduit par la Rédaction]

Introduction

Debris flows occur when masses of poorly sorted sediment, agitated and saturated with water, surge down slopes in response to gravitational attraction (Iverson 1997). Debris flows differ from rock avalanches and sediment-laden water floods because both solid and fluid forces influence the motions and govern rheological properties of the debris flows (Iverson 1997). Stresses caused by solid particle collisions, contact friction, and solid–fluid interactions generally govern the kinematic properties of debris flows. Fine particles (e.g., silts and clays) may be considered as pore-fluid components when the timescale required for viscous settling (without particle interactions) exceeds the flow duration (Iverson 1997; Iverson and Denlinger 2001). The apparent viscosity of the pore fluid in a debris flow is much larger than that of water. Meanwhile, coarse particles (e.g., gravels) interact during transport and significantly influence the kinematic properties of the debris flow.

Generally, constitutive laws for debris flows can be classified into two different types: macroscopic continuum treatment and microscopic analysis of particle interactions. The continuum representation of a granular material in terms of the conservation of mass and momentum has been the subject of extensive research for many years (MiDi 2004). However, some researchers argue that the solid phase shows a discrete behaviour and the presence of interstitial fluid significantly alters the behaviour of flows. One should separate the modeling of the fluid and granular materials in a unified Navier–Stokes approach (Iverson 1997; Iverson and Denlinger 2001).

No matter what method is applied to describe the kinematics of debris flows, identifying the flow regimes for the study of the rheological properties is always the most impor-

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tant issue. Bagnold (1954) reported that flows of solid–fluid mixtures can be divided into grain-inertial flows and macroviscous flows. These flows depend on the relative significance of solid collisions and pore-fluid viscosity. A dimensionless number called the Bagnold number (N_{Bag}) was developed to identify flow regimes. In grain-inertia flows (corresponding to a large N_{Bag}), particle collisions dominate. The momentum transfer and frequency of collisions are both proportional to the relative velocity difference between two neighboring layers. The corresponding stresses caused by collisions are proportional to the square of the shear rate. In macroviscous flows (corresponding to a small N_{Bag}), the effect of fluid viscosity is dominant. Shear and normal stresses inside granular bodies are linearly proportional to the shear rate. Obviously, analytical models for different flows are quite different. Dimensionless numbers are a good criterion for identifying flow regimes.

A qualitative interpretation of scaling terms and dimensionless numbers applied to describing physical model test results were presented by Iverson (1997). A fundamental knowledge about the microbehaviour of debris flows and the interactions between different phases was elaborated. As argued by Iverson and Denlinger (2001), the model scale effect plays a very important role in flow mechanisms. When the flow size increases, the importance of the viscous stresses diminishes and pore-fluid pressure effects grow more pronounced. Because most physical model tests were conducted at limited scales, the critical values of the dimensionless numbers obtained generally do not represent the essence of real, large debris flows. In addition, physical model tests generally simplify the flow conditions. The materials used in such tests are mixtures of uniform particles in clean water (e.g., Bagnold 1954; Hanes and Inman 1985). In contrast, natural debris flows include a wide range of particle sizes of 10^{-5} to 10 m (Iverson 1997). Interstitial fluid made up of water and fine suspensions (e.g., silts and clays) is usually nonNewtonian in behaviour. It is very difficult to determine a representative particle size to differentiate between the solid phase and the fluid phase in debris flows. It is also very difficult to determine the representative values of pore-fluid viscosity. For these reasons, dimensional analysis is applied to analyse large-scale natural debris flows. In this paper, the recorded field data of natural debris flows in Dongchuan, China, are reinterpreted based on dimensional analysis. The identification of different flow regimes and mechanisms for natural debris flows using dimensionless numbers is presented. In addition, the channel width as a confinement to flows is discussed.

Dimensional analysis

Parameters influencing the kinematic properties of debris flows

In general, debris flows are significantly influenced by slope geometry and intrinsic material properties. In a macroscopic view of the kinematics of debris flows, geometric factors, such as slope angle, θ, channel width, W, total volume of the debris flow, V, vertical drop height of the debris flow from the initiation point, H, and flow thickness, h, normal to the slope bed, should be considered. The properties of solid–fluid mixtures including water content, ω, volume fraction of the solid or fluid, C_S or C_f, respectively, and composite mixture stiffness of the debris, E, should be included in the analysis. To consider interactions between solid particles and between solid and fluid phases, intrinsic properties of the materials must be considered. These properties include mean particle diameter, d, mass density of the solid or fluid constituent, ρ_s or ρ_f, respectively, effective contact friction angle, ϕ', and viscosity of the pore fluid, μ_f. Other factors, such as gravitational acceleration, g, flow rate, Q, shear rate, $\dot{\gamma}$, granular temperature, T, pore-air pressure, u_a, and pore-water pressure, u_w, are also taken into consideration.

Dimensionless groups

To study the contact behaviour of solid particles and the effect of the pore fluid, a dimensional analysis is conducted and key dimensionless groups are formed to interpret the flow mechanisms. First, the traveling velocity of the debris flow, U, and the stress, σ, inside the granular body can be expressed by a function ψ

$$[1] \qquad (U, \sigma) = \psi(\theta, W, V, H, h, \omega, C_S, C_f, E, \dot{\gamma}, d, \rho_s, \rho_f, \phi', \mu_f, u_a, u_w, g, Q, T)$$

Second, and the most important step in the dimensional analysis of debris flows, is choosing independent primary dimensions for scaling the parameters in eq. [1]. Obviously, the parameters involved in eq. [1] have three basic dimensions: length, time, and mass. According to Buckingham's theorem (Buckingham 1914), these three basic parameters should be used to scale the other parameters. This paper focuses on the microcontact behaviour of solid particles. A characteristic length, d, a characteristic time, $1/\dot{\gamma}$, and a characteristic mass, $\rho_s d^3$, are chosen to be the three independent primary dimensions (Iverson 1997) for scaling the other parameters in eq. [1].

Take gravitational acceleration, g, as an example, which can be rescaled as $[g] = [\rho_s]^A[d]^B[\dot{\gamma}]^C$. Rewrite the dimensions of each parameter to have $[LT^{-2}] = [ML^{-3}]^A[L]^B[T^{-1}]^C$, which implies that $A = 0$, $B = 1$, and $C = 2$. Thus, the gravitational acceleration, g, is scaled by $\dot{\gamma}^2 d$ and a dimensionless group, $g/\dot{\gamma}^2 d$, is constructed. With the same approach, the other parameters can be scaled by different combinations of ρ_s, d, and $\dot{\gamma}$. Some dimensionless parameters, such as the contact friction angle, ϕ', slope angle, θ, and water content, ω, are intrinsically dimensionless and have identical effects on different scales.

The scaled traveling velocity of the debris flow can then be expressed by a new function, f, where only dimensionless groups are involved

$$[2]\qquad \left(\frac{U}{\dot{\gamma}d},\ \frac{\sigma}{\rho_s\dot{\gamma}^2d^2}\right)=f\left(\frac{g}{\dot{\gamma}^2d},\ \frac{\mu_f}{\rho_s\dot{\gamma}d^2},\ \frac{h}{d},\ \frac{W}{d},\ \frac{T}{\dot{\gamma}^2d^2},\ \frac{H}{d},\ \frac{E}{\rho_s\dot{\gamma}^2d^2},\ \frac{\rho_f}{\rho_s},\ \frac{(u_a-u_w)\omega}{\rho_s\dot{\gamma}^2d^2},\ \frac{V}{d^3},\ \frac{Q}{\dot{\gamma}d^3},\ \frac{\tan\phi'}{\tan\theta}\right)$$

Equation [2] shows that multiple dimensionless numbers can affect the traveling velocity and stress levels inside the granular body. The relative significance of the dimensionless numbers is different. Normal stresses taking solid concentration and flow thickness into account govern the contact friction between solid layers and then influence the flow mobility. The physical meanings of dimensionless numbers and their influence on normal stresses acting on the slope bed for natural debris flows are elaborated in this paper. Compared with Iverson's (1997) work, geometrical properties (e.g., slope angle, channel width, and drop height of the debris mass) are considered in eq. [2]. Thus, both macro and micro behaviour are involved in the dimensional analysis. In addition, the pore-air and pore-water pressures are taken into account. In unsaturated cases (usually the front head of the debris flow), a matric suction item may have a significant effect on flow mobility (Iverson 1997). Therefore, a suction term — $(u_a - u_w)\omega/\rho_s\dot{\gamma}^2d^2$ in eq. [2] — is also involved in the dimensional analysis. It generally decreases to zero when the debris flows are fully saturated.

Field study in the Dongchuan Debris Flow Observation and Research Station

Location of field study

The Dongchuan Debris Flow Observation and Research Station (DDFORS), located in Dongchuan City in the Yunnan Province of China (N26°14′, E103°08′) and built in the 1960s, is a facility of the Institute of Mountain Hazards and Environment, Chinese Academy of Science (Cui et al. 2005). This is the sole semi-automatic field observation station of debris flows in China. Continual observations of field debris flows that have occurred in the Jiangjia Ravine have been conducted since the construction of the DDFORS. Debris flow initiation, transportation, and deposition have been monitored and forecasting, warning, and mitigation of debris flows have also been attempted.

As reported by Cui et al. (2005), the Jiangjia Ravine is located on the east side of the Xiaojiang River and developed in the area of the Xiaojiang fault (see Fig. 1). The area of the Jiangjia Ravine is about 48.6 km^2 and is characterized by intense tectonism. About 80% of the exposed rocks are highly fractured and mildly metamorphosed. Both the sandstone and the slate are weak and easily weathered and broken into fragments. Colluvium and mantle rock are widely distributed on the slopes and in the channels in some sub-basins. Accumulated clastic detritus is the main source of the materials in the debris flows when an intense rainfall occurs. In fact, most of the recorded debris flows in China are triggered by rainfall. The annual rainfall in the Jiangjia Ravine ranges from 400–1000 mm (Cui et al. 2005) but is marked as being seasonal. About 85% of the total annual rainfall occurs between May and October. These conditions provide an ideal setting for recurrent debris flows and 10–20 debris flows occur there annually (maximum 28 debris flows). The ravine is now called the "debris museum" in China and supplies advantageous research conditions.

Measurements at the DDFORS

The measurements at the DDFORS include front velocity, U (m/s), channel width, W (m), traveling thickness of debris flow, h (m), unit weight of each surge, γ_c (kg/m^3), and duration of each surge, t (s).

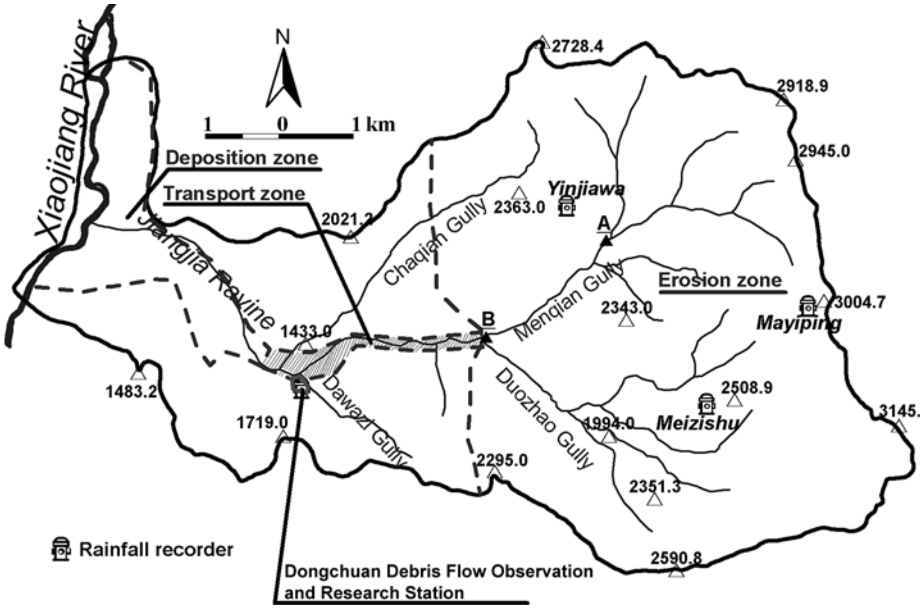
Two monitoring cross sections are marked along a straight channel, and the distance ΔL used to measure the velocity of the surge front has been predetermined. For each debris flow, the time for the surge front to pass through ΔL is recorded by a stopwatch and marked as Δt (Zhang and Xiong 1997; Kang et al. 2006; Kang et al. 2007). The mean front velocity is then calculated as $U = \Delta L/\Delta t$. Referring to the cross-section marks, the channel width, W, and the ground level for each debris flow can be determined. With a supersonic lever meter installed above the front head, the flow thickness, h (m), can be measured. In addition, the front discharge, Q, can be calculated as $Q = UWh$. Through direct sampling of debris flows, the unit weight of each surge, γ_c (kg/m^3), is determined. Thus, the solid volume concentration, C_S, can be calculated from $C_S = (\gamma_c - \gamma_w)/(G_S - \gamma_w)$ (where γ_w and G_S are the unit weights of water and the solid material, respectively, and G_S is generally taken to be 2750 kg/m^3). The solids discharge, Q_S (t/s), is then calculated as $Q_S = QG_SC_S/1000$. Furthermore, as the duration of each surge t is recorded, the total (or sediment) runoff of each surge can be determined. The field observation data from 1961 to 2000 were collected and edited by Zhang and Xiong (1997), Kang et al. (2006), and Kang et al. (2007). From these data, those of the first debris flow occurring during each year from 1990 to 1995 were chosen for analysis.

Study of observations at the DDFORS

Based on long-term observations at the DDFORS, debris flows in the Jiangjia Ravine can be classified into two different types: continuous flows and surge flows. Generally, by empirical estimation, either the flow time of a continuous flow is quite long or the discharge is quite large. If there is an obvious flow break between two continual surges or the surge duration is relatively short, the flow is treated as a surge flow (Zhang and Xiong 1997; Kang et al. 2006; Kang et al. 2007). Usually, a debris flow is composed of dozens of surge flows and a few continuous flows.

Figure 2 shows the relationships between the recorded front discharge, Q, and the solids discharge, Q_S, for debris flows from 1990 to 1995. The solid volume concentration can be calculated from $C_S = Q_S/(QG_S)$. At the DDFORS, C_S of the surge flows is between 0.3 and 0.72; 0.6 on average. In continuous flows, C_S is between 0.17 and 0.68; 0.45 on average. Compared with hyperconcentrated flows ($C_S \le 0.6$) (Chien and Wan 1999; Mulder and Alexander 2001), the

Fig. 1. Plan of Jiangjia Ravine; numbers are elevations in metres above sea level (from Cui et al. 2005, with permission).

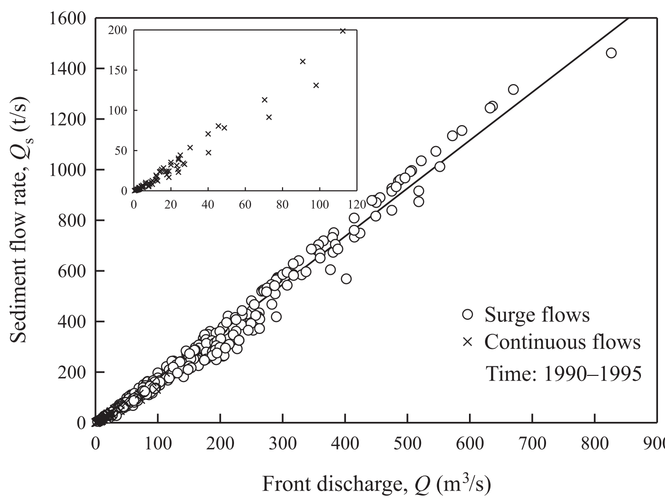


solid concentration of surge flows is relatively high. The flow behaviour of surge flows is significantly governed by solid particle interactions. In contrast, C_S for continuous flows is quite low. Solid particles are diluted in the viscous fluid and their behaviour is more likely governed by fluid flows. Figure 2 shows that for different debris flows at the DDFORS, the solids discharge, Q_S, increases linearly with the front discharge, Q. The highly convergent data points on a linear curve for surge and continuous flows are observed. The solid volume concentrations, C_S (the slope of the curves), stay almost constant for surge and continuous debris flows, i.e., independent of the front discharge, Q. This implies that the sediment transport capacity is saturated for natural debris flows.

It is a commonly accepted concept that a debris flow consists of two phases: the solid phase and the fluid phase. The fluid phase is a mixture composed of pore fluid and fine particles. A useful, but inexact, guideline proposed by Iverson (1997) states that grains larger than silt size comprise the debris flow solids, whereas grains in the silt–clay fines fraction act as part of the fluid. This implies that pore-fluid viscosity is significantly influenced by fine particles, but to date this research still remains rudimentary. Through field samplings and ring shear tests in a coaxial viscosimeter (by Cui et al. 2005), an approximated dynamic viscosity (0.5 Pa·s) of the pore-fluid phase (water and solid particles of $d \leq 2$ mm) is suggested for the analysis of natural debris flows in the Jiangjia Ravine. This is much larger than the viscosity for debris flows suggested by Iverson (1997), which is in the range of 0.001–0.1 Pa·s. According to field observations and classifications by Fei et al. (1991) and Kang et al. (2004), particles of $d > 2$ mm may be considered as the solid phase, whereas solid particles of $d \leq 2$ mm are treated as the pore-fluid phase.

For surge flows, material gradation varies with position. Although the density of the front head is larger than that of the rest of the flow because the coarsest clasts concentrate there, the difference in density is usually not very significant, i.e., the particle-size distributions within the surge are quite similar (cf. Cui et al. 2005). Based on the particle-size distributions for the samples taken from field debris flows in the Jiangjia Ravine at different periods in time (see Fig. 3), a mean particle size d_{50} (= 5 mm) has been chosen for the analysis in this paper (Cui et al. 2005; Kang et al. 2006). With these parameters, the dimensionless numbers in eq. [2] are determined and the analysis is described below.

Fig. 2. Field correlation between front discharge and solids discharge.



Study of dimensionless groups

In the following analysis of the field data, a simple shear model is applied (see Fig. 4). It assumes that, along the flow depth, the maximum velocity, $U_{\max}$, is at the free surface and the traveling velocity is zero at the slope bed. The shear rate is defined as the velocity gradient normal to the traveling direction. A mean shear rate is calculated directly as a ratio of the free surface velocity divided by the traveling height (= $U_{\max}/h$), where the magnitude of $U_{\max}$ is approxi-

Fig. 3. Particle-size distribution of the natural debris flows at the DDFORS.

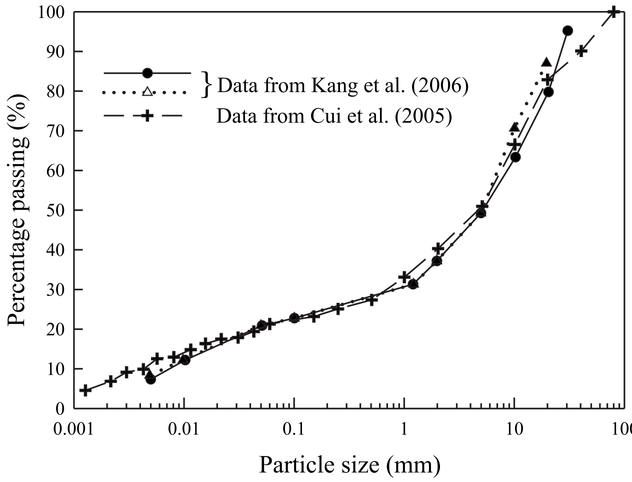
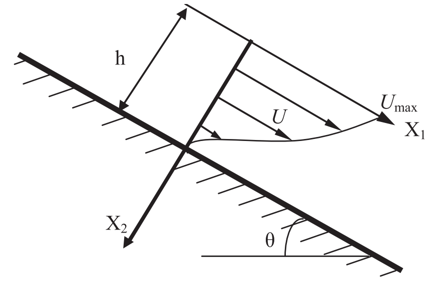


Fig. 4. Two-dimensional gravity flow of granular material down an inclined plane (from Savage 1984, with permission).



mately equal to the measured front velocity, U. This simple shear model is similar to the simple viscometric flow model applied by Savage (1984) to the two-dimensional steady free-surface flow down a rough inclined plane. Solid particles submerged in the flows are assumed to have the same shear rate as the pore fluid. It should be noted that the relative movement of grains and the pore fluid is assumed to be controlled only by the linear Stokes (or Darcy) relationship. Nonlinearities, such as yield strength, varying viscosity or turbulence, are not considered in this paper.

Modified Savage number in debris flows

The dimensionless number $g/\dot{\gamma}^2 d$ in eq. [2] was described by Savage (1984) as a parameter for identifying flow regimes of dry granular flows. It was later called the Savage number. A further modification to the Savage number was proposed by Iverson (1997). It includes the effective contact shear stress and the solid inertial stress among solid particles in fully saturated granular flows.

The Savage number modified by Iverson is defined as

$$[3] \qquad N_{\mathrm{Sav}} = \frac{T_{\mathrm{s(i)}}}{T_{\mathrm{s(q)}}} = \frac{\rho_{\mathrm{s}} \dot{\gamma}^2 d^2}{(\rho_{\mathrm{s}} - \rho_{\mathrm{f}}) g h \cos\theta \tan\phi'}$$

The solid inertial stress, $T_{s(i)}$, in eq. [3] is caused by particle collisions. According to Bagnold (1954), this stress is proportional to the square of the shear rate and the particle diameter. It may be estimated by $T_{s(i)} \sim C_S \rho_S \dot{\gamma}^2 d^2$. Based on the Coulomb's law, the quasi-static solid stress, $T_{s(q)}$, due to particle contact friction may be estimated from $T_{s(q)} \sim C_S(\rho_S - \rho_f) g h \cos\theta \tan\phi'$. As described by eq. [3], the physical meaning of the modified Savage number, N_{Sav}, is the ratio of the solid inertial stress to the quasi-static contact shear stress. This number accounts for the relative significance of the enduring contact friction and the collisions among particles. When N_{Sav} is relatively large, the contact shearing of solid layers is less dominant. The kinematic properties of granular flows are influenced mainly by particle collisions. Through annular-shear-cell tests of dry granular materials (Savage and Sayed 1984) and granular–fluid mixtures (Hanes and Inman 1985), an approximate guide was proposed by Savage and Hutter (1989). If N_{Sav} is larger than 0.1, the grain inertial stress dominates the contact friction stress in granular flows. In contrast, a small N_{Sav} indicates that collisions likely transmit a low amount of stress in such flows and the contact friction may dominate.

If the contact friction angle is assumed to be 30° for granular materials, the Savage number of natural debris flows at the DDFORS may be calculated. Additionally, based on the measured bulk density, ρ (= $\rho_f + C_S(\rho_s - \rho_f)$), and the flow thickness, h, the normal stress, σ, acting on the slope bed is calculated from $\rho g h \cos\theta$. Figure 5 shows a correlation between the normal stress, σ, and the modified Savage number, N_{Sav}.

Figure 5 shows that all the modified Savage numbers, N_{Sav}, for surge and continuous flows are smaller than 0.1. As suggested by Savage and Hutter (1989), the natural debris flows should be treated as contact friction–dominated flows. The momentum transfer through particle collisions plays a relatively less important role for natural debris flows. Figure 5 also illustrates that the normal stress acting on the slope bed decreases when the modified Savage number, N_{Sav}, increases. As N_{Sav} is directly related to the shear rate in eq. [3], the stress level inside the debris body is significantly dependent on the shear rate. Although the contact friction is independent of the shear rate and dominates the particle collisions among solid particles, the natural debris flows possess obvious rheological properties.

Similar trends are observed for surge and continuous flows: the normal stress on slope bed reduces as N_{Sav} increases. It is postulated that an increase in the shear rate, $\dot{\gamma}$, causes the solid inertial velocity $\dot{\gamma}d$ to increase, and the flow thickness, h, is expected to be greatly reduced accordingly. Thus, the total normal stress based on the calculation of $\rho g h \cos\theta$ is reduced. Dragged by the flowing fluid, contact between solid particles tends to become loose. Then, a large N_{Sav} is induced and the flow tends to be dominated by solid collisions. Although the solid concentration of the continuous flows is relatively small (0.17–0.68 and 0.45 on average) and the particles tend to be diluted in fluids easily, all of the continuous flows have a small N_{Sav} (<0.1) and contact friction still dominates the flows. A possible explanation is that the velocity gradient that develops in the channelized flows is greatly limited (<32 1/s) compared with that in the ring shear tests (e.g., Savage and Sayed 1984).

Grain Reynolds number in debris flows

In fluid mechanics, the Reynolds number is well known for identifying laminar and turbulent flows. For grain–fluid mixture flows, the grain Reynolds number, N_{Rey}, is applied

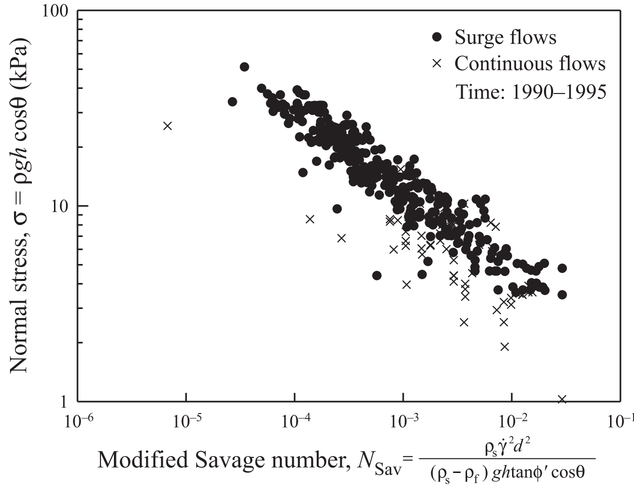


Fig. 5. Normal stress in relation to modified Savage number, N_{Sav}, in natural debris flows.

to identify the effects of particle collisions and pore-fluid viscosity. The definition of the grain Reynolds number is given in eq. [4] (cf. Iverson 1997)

$$[4] \qquad N_{Rey} = \frac{T_{s(i)}/T_{f(q)}}{\frac{C_S}{1-C_S}\frac{\rho_s}{\rho_f}} = \frac{\rho_f \dot{\gamma} d^2}{\mu_f} = \frac{d^2}{\delta_v^2}$$

which is quite similar to the second item on the right side of eq. [2]. Equation [4] clarifies the physical meaning of the grain Reynolds number, N_{Rey}: it is a ratio between the solid inertial stress and the fluid viscous shearing stress. The solid inertial stress may be estimated by $T_{s(i)} \sim C_S \rho_s \dot{\gamma}^2 d^2$ and the fluid viscous shearing stress derived from the Newton's law of viscosity is $T_{f(q)} = C_f \dot{\gamma} \mu_f$. Also, the grain Reynolds number, N_{Rey}, can be considered as a ratio of the solid particle size, d, to the viscous diffusion length, δ_v, as illustrated by Hsu.[2]

Figure 6 (from Hsu,[2] with permission) shows a potential flow traveling over a solid particle, where δ_v is the boundary layer thickness (viscous diffusive length) around the solid surface and L represents the particle size. Inside the boundary layer, a laminar flow is formed and the properties are governed by the fluid viscosity. Consistent with eq. [4], the grain Reynolds number can be defined as $N_{Rey} = L^2/\delta_v^2$ (here, the particle size L is generally chosen to be the particle diameter d, and the viscous diffusion length δ_v in the advection time interval of $1/\dot{\gamma}$ is $(\mu_f/\rho_f\dot{\gamma})^{1/2}$). When the grain Reynolds number is high (e.g., $N_{Rey} \gg 1$), the solid inertial stress is much larger than the fluid viscous shearing stress. The viscous diffusion distance, δ_v, is much smaller than the length scale, L. Then the viscous drag on the solid surface has a negligible effect. In contrast, $N_{Rey} \ll 1$ means that the solid inertial stress is much smaller than the fluid viscous shearing stress or the viscous diffusion length, δ_v, is much larger than L. Thus, the particle is dragged by the viscous force of the fluid and its movement is governed mainly by pore-fluid flows.

The correlation between the total normal stress and the grain Reynolds number is shown in Fig. 7 for the case

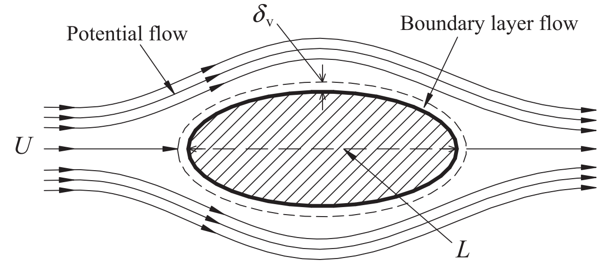


Fig. 6. Fluid flow around a solid particle for high grain Reynolds number flows.

where an approximated viscosity based on tests (μ = 0.5 Pa·s) is chosen for the debris flows at the DDFORS (Cui et al. 2005). Obviously, most of the grain Reynolds numbers, N_{Rey}, are smaller than 1. This implies that the solid inertial stress is less dominant than the fluid viscous shearing stress or that the diffusive distance, δ_v, is relatively larger than the solid grain size. It also indicates that, for natural debris flows, fluid viscous shearing has a more significant effect on the mobility of the debris flows than do solid collisions. In contrast, if the pore-fluid viscosity is small (e.g., 0.05 Pa·s as suggested by Iverson (1997)), N_{Rey} is significantly larger than 1 and the particle inertial stress may be dominant. In addition, when the grain Reynolds number is reduced, the normal stress acting on the slope bed increases and the roles played by viscous drag forces on the solid particles are increasingly important.

Normal stress on slope bed

In Figs. 5 and 7, the normal stress variations according to N_{Sav} and N_{Rey} indicate that the total normal stress acting on the slope is influenced by the shear rate, pore-fluid viscosity, and particle concentration. Bagnold (1954) investigated the stress state for wet granular flows and defined the solid inertial stress caused by particle collisions. As particle collision frequency and the momentum transfer between contact particles are both correlated to the shear rate, the solid inertial stress (dispersive stress) is proportional to the square of the shear rate and linear grain concentration.

The correlation of the normal stress and the shear rate in natural debris flows is shown in Fig. 8. (For simplicity, in Figs. 8 and 9 the prefix "S" represents surge flows and "C" signifies continuous flows. For example, the surge flows of the first debris flow observed in 1990 are designated "S9001".) The normal stresses acting on the slope bed are mostly proportional to $\dot{\gamma}^{-1}$. In addition, Fig. 9 shows the correlation between the normal stress and the solid inertial stress, $T_{s(i)}$, due to particle collisions. The normal stress is not only dependent on $\dot{\gamma}^{-1}$ but is also correlated to $1/(C_S)^{1/2}$, and the changing trend is contrary to Bagnold's results.

Referring to Bagnold's (1954) tests using an annulus shear cell, one can find that the flow thickness and the solid concentration are fixed. The shear rate is controlled as a constant in granular flows with a high solid concentration. Normal stresses on the side walls increase synchronously with the shear rate. However, natural debris flows are free

[2] Hsu, C.T. Foundation of fluid mechanics. Book in manuscript stage.

Fig. 7. Correlation of normal stress and grain Reynolds number, N_{Rey}.

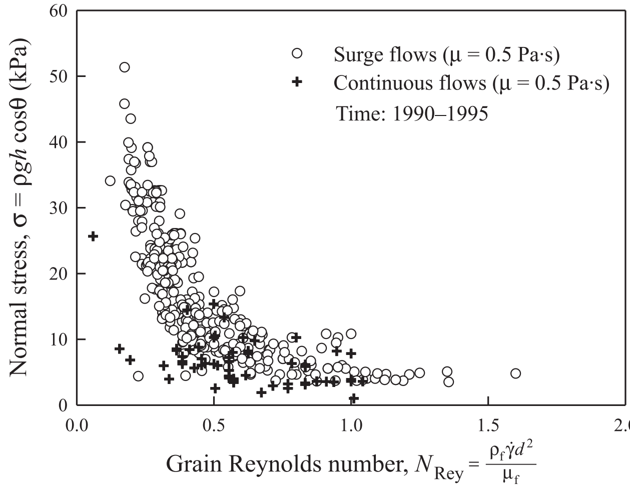


Fig. 8. Correlation of normal stress and shear rate.

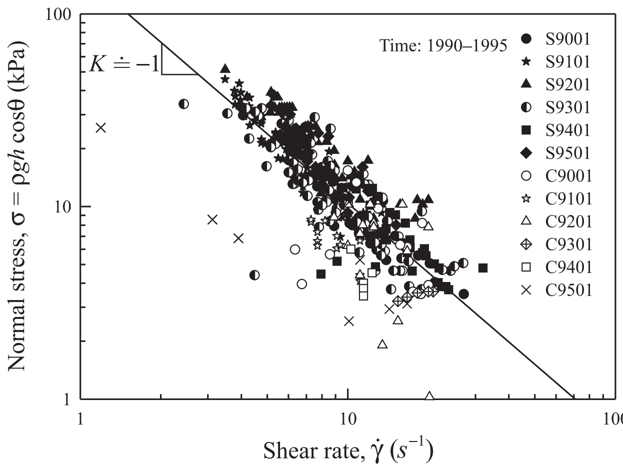
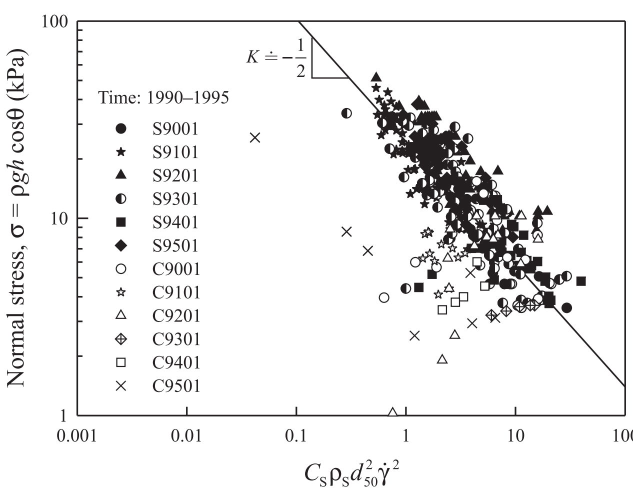


Fig. 9. Comparison of normal stress and solid inertial stress by collisions.



surface flows. Except for the slope bed, there is no normal confinement to debris flows. The shear rate inside the granular mass can adjust according to the flow thickness and descending velocity. An increase in shear rate is generally followed by an elongated flow length and reduced flow thickness of debris flows. This may cause the total normal stress acting on the slope bed to decrease accordingly.

Friction number in debris flows

When comparing the relative effects of particle contact friction and pore-fluid shearing, a friction number is necessary (Iverson 1997)

$$[5] \qquad N_{\mathrm{fric}} = \frac{C_{\mathrm{S}}}{1 - C_{\mathrm{S}}} \frac{N_{\mathrm{Rey}}}{N_{\mathrm{Sav}}} = \frac{C_{\mathrm{S}}}{1 - C_{\mathrm{S}}} \frac{(\rho_{\mathrm{s}} - \rho_{\mathrm{f}}) g h \cos\theta \tan\phi'}{\dot{\gamma} \mu_{\mathrm{f}}}$$

This number expresses the ratio of the shear stresses caused by enduring grain contacts and pore-fluid viscous shearing. Large values of the friction number suggest that the solid frictional shear stresses probably exceed the pore-fluid viscous shearing stresses. Figure 10 shows that the influence of N_{fric} on the normal stresses is similar for surge and continuous flows. An increase in N_{fric} results in an increase in the normal stress and different types of natural debris flows are unified on a curve. A large value of N_{fric} (mostly larger than 100) means that shear stress from solid contact shearing is larger than that from pore-fluid viscous shearing in most natural debris flows. When the pore-fluid viscosity or shear rate is increased, the pore-fluid viscous shearing stress, $T_{\mathrm{f(q)}} = C_{\mathrm{f}} \dot{\gamma} \mu_{\mathrm{f}}$, will increase accordingly. Then the friction number, N_{fric}, is reduced and the shear stress by pore-fluid viscous shearing will take on a significant role in determining the rheological properties of debris flows. Compared with surge flows, continuous flows have low solid concentrations. Diluted solid particles submerged in a viscous fluid are more easily dragged by the viscous shearing stress. Consequently, smaller friction numbers are found for continuous flows.

Effect of channel confinement on the solids discharge per unit width

Dynamic properties of debris flows are not only influenced by interactions between phases but also by slope and channel geometries. In systematic studies, channel confinements should be taken into consideration. With the same method developed by Jop et al. (2005) for steady uniform flows (dry dense granular flows), the normalized solids discharge per unit width can be defined as $Q_{\mathrm{S}}^{*} = QC_{\mathrm{S}}/[Wd_{50}(gd_{50})^{1/2}]$, the normalized channel width as $W^{*} = W/d_{50}$, and the normalized flow thickness as $h^{*} = h/d_{50}$. Figure 11 shows that consistent with the results from dry dense granular flows modeled by Jop et al. (2005), surge and continuous debris flows are influenced by the channel confinement. As demonstrated by Jop et al. (2006), the flow thickness, h, scales with the channel width, W, meaning that neither the thickness of the flow layer nor the shear rate are intrinsic properties of the granular media. Indeed, granular flows are controlled by the width of the channel and the solids discharge (per unit width). In addition, with the same solids discharge and under the same channel confinement, dry granular flows have a slightly higher flow thickness. This implies that the pore-fluid viscosity creates resistance to the dilution of the granular materials when particle contact shearing and collisions develop in the granular mass. In dry dense granular flows, a correlation between Q_{S}^{*} and $h^{*}/$

Fig. 10. Correlation of normal stress and Friction number, N_{fric}.

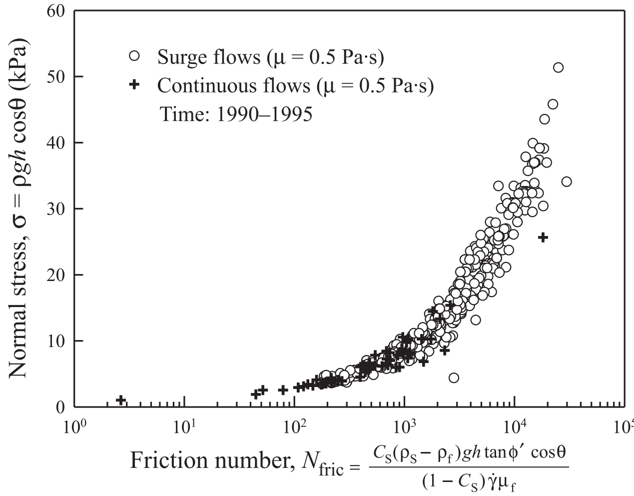
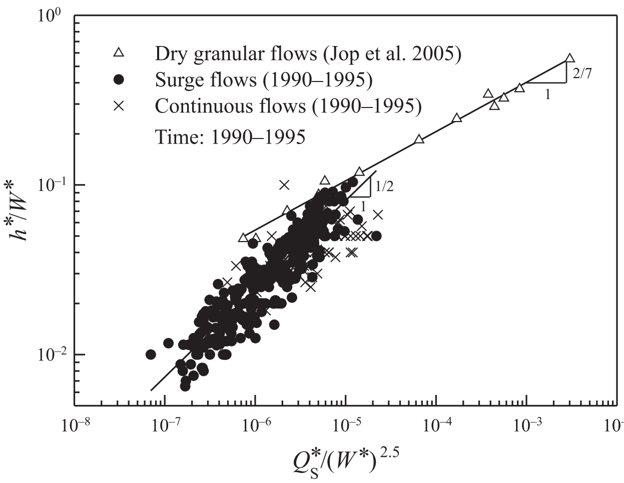


Fig. 11. Rescaled traveling thickness in relation to rescaled solids discharge per unit width.



W^* was obtained by Jop et al. (2005). The theoretical model can predict the results of the physical model tests quite well and prove that $(h^*/W^*) \sim [Q_S^*/(W^*)^{2.5}]^{2/7}$. However, this relationship should be changed to $(h^*/W^*) \sim [Q_S^*/(W^*)^{2.5}]^{1/2}$ for natural debris flows, as illustrated in Fig. 11. Further modification of these equations shows that

$$[6] \qquad \frac{\partial Q_S^*}{\partial W^*} \sim (-1)(h^*)^{7/2}(W^*)^{-2} < 0$$

for dense granular flows and

$$[7] \qquad \frac{\partial Q_S^*}{\partial W^*} \sim (h^*)^2(W^*)^{-0.5} > 0$$

for natural debris flows. This implies that the increase of channel width will increase the solids discharge per unit width for natural debris flows (see eq. [7]), and it is quite different from uniform granular flows (see eq. [6]).

Classification of natural debris flows

To date, the classification of natural debris flows is generally based on empirical observations. Few clear criteria are currently available to clarify the different types of natural debris flows. For instance, the natural debris flows at the DDFORS were classified into surge and continuous flows based on the surge duration by empirical observations. Generally, the surge duration is quite different for surge and continuous flows. However, occasionally the time difference is very small and it is quite difficult to identify whether the surge duration is long or short. Usually, mistakes are caused by the empirical judgements. For this reason, a uniform criterion that has clear physical meanings needs to be developed to clarify different natural debris flows.

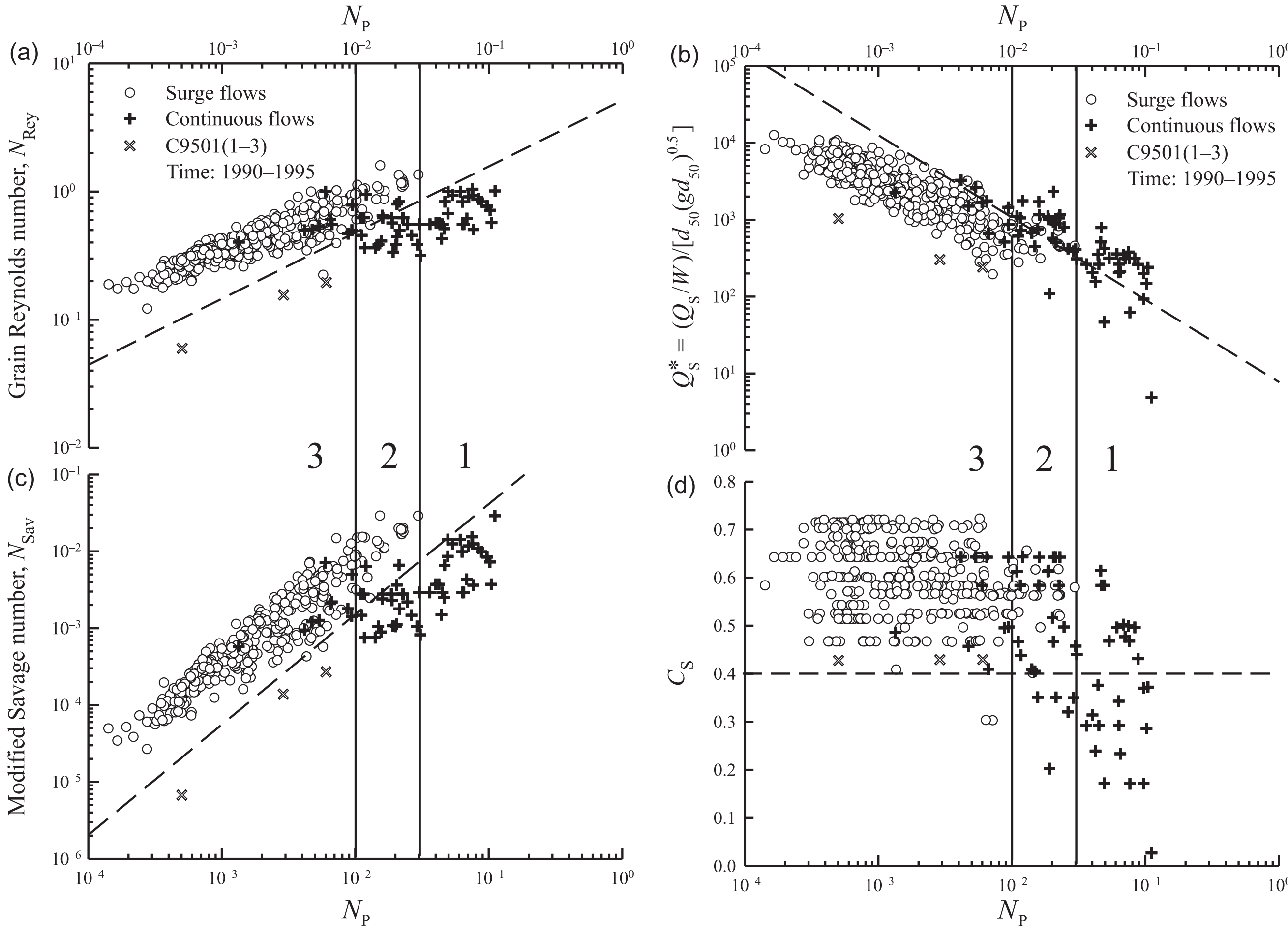
Figures 5, 7, and 10 show that the contact behaviour for solid particles in surge and continuous flows is generally unified and can be identified by the three dimensionless numbers. The channel confinement effects on these flows are almost the same. To distinguish the different debris flows based on field observation data, a new criterion correlating with the surge duration, t, needs to be developed. Reconsider the dimensionless number N_p defined by Iverson et al. (2004)

$$[8] \qquad N_p = \frac{\sqrt{l/g}}{\mu_f h^2/kE}$$

where l is the avalanche length that can be calculated by Ut and k is the hydraulic permeability. The definition shows that N_p is not only correlated to surge duration but also has clear physical meanings. Indeed, this number is the time-scale ratio of avalanche motion $(l/g)^{1/2}$ and the diffusion of disequilibrium pore-fluid pressure normal to the flow direction ($\mu_f h^2/kE$). Small values of N_p indicate that high pore-fluid pressure can be sustained for quite a long duration inside a granular body and influence the avalanche motion significantly (Iverson et al. 2004). With a suggested hydraulic permeability ($k = 10^{-11}$ m^2) and a compressive stiffness ($E = 10^7$ Pa) for typical loose granular soils (Iverson 1997; Iverson et al. 2004), the N_p values for surge and continuous debris flows can be calculated. The correlation between N_p and contact behaviour of solid particles in natural debris flows is shown in Figs. 12*a* and 12*c*: the grain Reynolds number, N_{Rey}, and the modified Savage number, N_{Sav}, increase with N_p. This implies that when solid particles have strong collisions and the debris flows tend to be inertial flows, pore-fluid pressure in a granular body will dissipate quickly and has little influence on particle contact behaviour. Furthermore, the solids discharge per unit width, Q_s^*, will be reduced accordingly as N_p increases (see Fig. 12*b*).

Generally, the N_p values of natural debris flows at the DDFORS can be classified into three different zones. Most of the continuous flows possess high values of N_p and are focused in zone 1 ($N_p > 0.03$). In contrast, most of the surge flows have relatively small values of N_p and are focused in zone 3 ($N_p < 0.01$). Figures 12*b* and 12*d* show that surge flows usually possess relatively higher solids discharge per unit width and larger solid concentrations than continuous flows. Due to the high solid concentration and the large traveling velocity for surge flows, high pore pressures are more likely developed in these flows. Relatively small N_p values for surge flows indicate that the high pore-fluid pressures can be sustained for quite a long duration. Compared with surge flows, continuous flows with diluted solid particles possess high N_p values. This implies that high pore

Fig. 12. Classification of natural debris flows as a function of dimensionless numbers: (*a*) relationship between N_p and grain Reynolds number, N_{Rey}; (*b*) relationship between N_p and normalized solids discharge per unit width, Q_s^*; (*c*) relationship between N_p and modified Savage number, N_{Sav}; and (*d*) relationship between N_p and solid concentration, C_S.



pressures developed inside the granular body may dissipate quickly and the effects of pore pressure are likely negligible.

It is also obvious in Fig. 12 that an overlapping zone exists: in zone 2 ($0.01 < N_p < 0.03$), continuous and surge debris flows have the same N_p values. Some flows in zone 2 are very difficult to identify as they possess both continuous and surge flow properties. For this reason, zone 2 can be treated as a transition zone. Based on the variation of N_p values with the dimensionless numbers (N_{Rey}, N_{Sav}, and Q_s^*) in zones 1 and 3, the boundaries (dashed lines) distinguishing continuous and surge flows are approximately determined. Different debris flows in zone 2 can be classified by the dashed lines (see Figs. 12*a*–12*c*). For instance, by comparing the flows in zones 1 and 3 of Figs. 12*a* and 12*c*, the flows located above the dashed lines can generally be treated as surge flows. Comparatively, the flows located below the dashed lines are treated as continuous flows. By this method, the classification of natural debris flows can be obtained with little ambiguity as different flows are focused in different zones.

Due to the limitations of observation and subjective judgment, some flows originally treated as "continuous flows" are indeed located in zone 3 above the dashed lines (see Figs. 12*a*, 12*c*). In these flows, Figs. 12*a*–12*c* show that the solid contact behaviour, solids discharge per unit width, and relative significance of pore pressure developed in the granular body are quite similar to that of surge flows. In addition, Fig. 12*d* shows that the solid concentrations of these flows are sometimes relatively high (>0.6). These "continuous flows" should be considered as surge flows. It is also interesting that the first three "continuous flows" occurring in 1995 (C9501(1–3)) are located in zone 3 below the dashed lines. Compared with surge flows, the surge durations of these flows are relatively long and the flow velocity is relatively low. Similar to surge flows, the timescale of avalanche motion for these flows is relatively smaller than the timescale for diffusion of disequilibrium pore-fluid pressure. The small N_p values imply that pore pressure inside the granular body may play a significant role. However, the contact behaviour of these flows is quite different from surge flows (see Figs. 12*a*, 12*c*). The solids discharge per unit width and the solid concentration are lower than that of surge flows (see Figs. 12*b*, 12*d*). Also, these flows are obviously different from most of the continuous flows that possess large N_p values. These special "continuous flows" may be classified as a new type of flow in the field, and the flow properties need to be further investigated in future work.

Conclusions

Dimensional analysis is a useful tool for systematic analysis of the factors that influence debris flows. By comparing different values of dimensionless numbers, including N_{Sav}, N_{Rey}, N_{fric}, and N_p, the rheological properties of debris flows can be predicted. Based on the field data collected at the Dongchuan Debris Flow Observation and Research Station (DDFORS), the properties of large-scale natural debris flows occurring in the field can be characterized as follows:

(1) For small values of N_{Sav} (<0.1) in natural debris flows at the DDFORS, flows are dominated by particle contact friction rather than by particle collisions. A collision-dominated trend (increase in N_{Sav}) tends to reduce the total normal stress acting on the slope surfaces.
(2) For small values of N_{Rey} (<1) in natural debris flows at the DDFORS, the fluid viscous shearing stress dominates solid inertial stress. In contrast to the effect of grain collisions, a pore-fluid viscous shearing–dominated trend (decrease in N_{Rey}) tends to increase the normal stress acting on the slope.
(3) For natural debris flows, the total normal stress acting on the slope bed, σ, is proportional to the reciprocal value of the shear rate ($\dot{\gamma}^{-1}$) and is also correlated to the solid concentration as $\sigma \sim (C_S)^{-1/2}$.
(4) Large values of N_{fric} ($\gg 1$) are found in natural debris flows. This implies that the shear stress borne by sustained grain contacts is larger than the fluid viscous shearing stress. A contact friction–dominated trend (increase of N_{fric}) tends to increase the normal stress acting on the slope bed. A consistent change of normal stress with N_{fric} is also observed for surge and continuous flows.
(5) Similar to the uniform granular flows, the natural debris flows possess a certain relationship between the rescaled flow thickness and the rescaled solids discharge per unit width. The channel width as a confinement has a significant effect on the kinematic properties of debris flows. Unlike uniform granular flows, an increase in the normalized channel width, W^*, will cause the rescaled solids discharge per unit width, Q_s^*, to increase.
(6) Based on the dimensionless numbers N_p, N_{Sav}, N_{Rey}, and Q_s^*, continuous and surge debris flows can be identified. The surge flows with high solid concentrations have relatively small N_p values. This indicates that, for surge flows, the high pore-fluid pressures generated in a granular body dissipate quite slowly and may influence particle contact behaviour significantly. When the natural debris flows tend to be solid collision–dominated, pore-fluid pressure will dissipate quickly and play a relatively negligible role in solid contact behaviour.

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List of symbols

A, B, C indices of dimensions in a dimensionless group
C_{f} volume fraction of pore fluid
C_{S} volume fraction of solids
d particle diameter
d_{50} mean particle diameter
E debris composite mixture stiffness
G_{S} unit weight of solid material
g gravitational acceleration
H vertical drop height of debris mass
h debris flow thickness normal to slope bed
h^* normalized flow thickness
k hydraulic permeability
L particle size along flow direction
ΔL measuring distance between two monitoring cross sections
l avalanche length
N_{Bag} Bagnold number
N_{fric} friction number
N_{p} dimensionless number correlated to pore-fluid pressure dissipation
N_{Rey} grain Reynolds number
N_{Sav} modified Savage number
Q flow rate of debris flow or front discharge
Q_{S} solids discharge
Q_{S}^* normalized solids discharge per unit width
T granular temperature
$T_{\mathrm{f(q)}}$ fluid viscous shearing stress
$T_{\mathrm{s(i)}}$ solid inertial stress
$T_{\mathrm{s(q)}}$ quasi-static solid stress
t duration of each surge
Δt time for surge front to pass through ΔL
U front velocity of debris flow
U_{max} traveling velocity at the free surface
u_{w} pore-water pressure
u_{a} pore-air pressure
V total volume of debris flow
W channel width
W^* normalized channel width
$\dot{\gamma}$ shear rate
γ_{c} unit weight of each surge
γ_{w} unit weight of water
δ_{v} thickness of boundary layer around a solid particle's surface
θ slope angle
μ viscosity
μ_{f} viscosity of pore fluid
ρ bulk density of debris flow
ρ_{s} mass density of debris flow solid constituents
ρ_{f} mass density of debris flow fluid constituents
σ normal stress acting on slope bed
ϕ' effective contact friction angle
ω water content